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RELATIONS BETWEEN THE GROWTH OF MATHEMATICS AND ECONOMICS IN THE EIGHTEENTH AND NINETEENTH CENTURIES

Abstract

This paper examines the relations between the growth of mathematics and economics during the eighteenth and nineteenth centuries. We identify several factors that may have contributed to the retardation in the growth of mathematical economics before the twentieth century. Although several authors have studied the development of mathematical economics, to our knowledge no one has focussed on the parallel development of mathematics and economics and the interplay between these subjects. Our basic thesis is that many mathematical concepts indispensable to modern mathematical economics were unavailable to economists before the twentieth century, either because they were yet to be developed or because many of the founding figures of Political Economy were inadequately trained in mathematics.

I. INTRODUCTION

Students of the history of economic thought will generally agree that even though there is ample evidence of the use of mathematics in economics in the eighteenth and nineteenth centuries, the mathematization of economic theory is largely a twentieth century development. Such topics in mathematical economics as dynamic analysis, game theory, optimization techniques (e.g., linear and non-linear programming), axiomatic techniques in general equilibrium analysis, models of uncertainty, and especially econometrics--all of which are today considered an important part of every practicing economists' toolkit--were mostly developed in the present century.[1] Why did the mathematization of economics take so long? Is it because appropriate mathematical tools were unavailable to eighteenth and nineteenth century economists? Or is it because earlier economists were poorly trained in mathematics? Perhaps the answer lies in the intellectual climate of the times when such efforts were discouraged by the important members of the profession. This paper attempts to pursue some of these issues. We propose and investigate several hypotheses because no single factor seems to be entirely responsible for this development, or rather, non-development.

The adoption of mathematical methods in economics within a relatively short space of time by Leon Walras, Vilfredo Pareto, F. Y. Edgeworth and Irving Fisher suggest the conjecture that perhaps the "time was ripe" in the last two decades of the nineteenth century. While there may be some truth in such an explanation it can too easily

become a form of evasion, as is perceptively noted by Lubos Novy while commenting on simultaneous discoveries in mathematics

> The fact that those discoveries took place at the same time cannot be explained by the vague cliche that the time was ripe, because one then has difficulty explaining the lack of understanding for these results shown by other mathematicians of the time. (Novy, 2)

The example of mathematics is most pertinent because social and political influences are generally expected to have minimal influence on mathematicians. However the general philosophical milieu or Weltanschauung does influence our notions of what are permissible solutions to any given problem. Unless our account is to recount a steady stream of progress, it is important to note that even in mathematics "sometimes two (or even more) different concepts and explanations existed, among which it was very hard to decide, in spite of their incompatibility" (*op. cit.*, 5). It is not unreasonable to conjecture that philosophical presuppositions determine the particular course taken in the presence of such uncertainties. While our general examples are taken from various European sources, attempts to link up mathematical economics with the intellectual "spirit of the age" will be made only for Britain. Section II deals with four reasons for the early discouragement of mathematical economics: the early missteps; the negative reception of early attempts; the lack of mathematical competence and conceptual clarity among economists; and finally, the lack of suitably developed mathematics. Section III presents our conclusions.

II. CONTRIBUTING FACTORS

II.A. The Role Played by the Initial Missteps

One factor that may have played a role in retarding the application of mathematics to economics is the discouragement caused by several failed initial attempts in this direction. A "false beginning" especially when committed by the important members in a field may very well stunt the future growth of the field. A notable case of a false beginning for mathematical economics is that of Francis Hutcheson, who taught Adam Smith, and is believed to have greatly influenced Smith's early interest in economics. Hutcheson in 1720 published An Inquiry into the Original Ideal of Beauty and Virtue, in which he attempted to assess the morality of human actions by using mathematical expressions and axioms. He gave a set of axioms to define rationality. These include (1) the extent of desire for a good depends upon the amount of utility to be realized from the object (2) when the amount of 'good' from an object equals the amount of 'evil' all desire or aversion ends. To this sensible attempt to incorporate the language of mathematics in economics, Hutcheson added his early attempts to reduce ethics to moral Newtonianism.

> The moment of evil, produduced by any agent is, as the product of his Hatred into his ability or $\mu = H \times A$. (Scott, 31-32)

This decision to copy Newton as closely as possible led to an amusing parody by Laurence Sterne:

> Hutcheson, in his philosophic treatise on beauty, harmony and order, plus's and minus's you to heaven or hell, by algebraic equations--so that none but an expert mathematician can ever be able to settle his accounts with S. Peter--and perhaps S. Matthew, who had been an officer in the customs, must be called in to audit them. (quoted by Scott, loc. cit.)

In the fourth edition Hutcheson removed these premature attempts at quantification.

A second noteworthy example of the use of mathematics being strongly criticized by fellow members of the profession is the case of Canard, a controversial figure in the history of mathematical economics. In order to oppose the physiocratic view that all taxes fall on land, Canard, who was not an economist by training, committed himself to analyzing the origins of the wealth of nations. Canard's work was recognized by the prestigious French Academy, Institut National des Sciences et Arts. However, Canard and his work in mathematical economics were relentlessly attacked by many well-known professional economists. In a review of Canard's book published in the influential Edinburgh Review, Francis Horner wrote:

> (Canard) has only translated into a foreign language less readily understood, truths of which the ordinary enunciation is intelligible and familiar to all. We will not deny that some branches of political economy, especially those which relate to circulation, money and the analysis of price, admit of being treated with a precision which almost approaches mathematic exactness. But a subject may possess this precision without requiring or even admitting the symbolic representation of Algebra. (Horner, 56)

Among those critical of Canard and his techniques were such illustrious names as J. B. Say, Cournot, Walras, and Schumpeter.[2] This unfortunate incident may have damaged the cause of mathematization of economics by earning mathematical techniques the animosity of many prominent economists. Not only did Canard's specific use of mathematics come under criticism, but the entire approach of applying mathematics to deal with topics in political economy was attacked.

Perhaps we are overplaying this point. However, the history of various sciences shows that incidents like these involving controversies over irrelevant issues can have significant and lasting impact. Consider a famous example from mathematics--the unfortunate debate regarding prior invention of tge calculus between Newton and Leibnitz. This incident appears to have so poisoned the intellectual atmosphere of the time between England, Newton's country, and the rest of Europe, that for a long period, mathematicians in England ignored most of the developments in mathematics in the continent, causing English mathematics to fall behind considerably.

These early missteps may well have slowed down the adoption of mathematics till the early nineteenth century but the moderate and prestigious defense provided by William Whewell in 1829 should have indicated the constructive role mathematics had to play. Whewell begins by noting the paradigm of physics.

> We can easily imagine what would have been the result if men had, without the aid of consistent mathematical calculation, attempted to make a system of mechanical philosophy. There would have been three errors difficult to avoid. They might have assumed their principles wrongly; they might have reasoned falsely from them in consequence of the complexity of the problem; or they might have neglected the disturbing causes which interfered with the effect of the principal forces. (Whewell, 5)

It was the mathematical formulation of physics that permitted one to avoid such avoidable errors.

> And the making mechanics into a Mathematical science supplied a remedy for all these defects. It made it necessary to state distinctly the assumptions, and these thus were open to a thorough examination; it made the reasonings almost infallible; and it gave results which could be compared with practice so as to shew whether the problem was approximately solved or not. It appears I think that the sciences of

> Mechanics and Political Economy are so far analogous, that something of the same advantage may be looked for from the application of mathematics in the case of Political Economy. (Whewell, 6)

Why were these sensible arguments not eagerly seized upon by economists?

II.B. The Negative Influence of the Prevailing Intellectual Climate

One of the factors that may have contributed to the retardation in the mathematization of economics before this century was the negative response given to initial attempts in this direction. There has always been a vocal group in the ranks of the profession itself that has resisted the use of mathematical tools in economic analysis. There is some evidence to suggest that, prior to the twentieth century, this group was both influential and effective in its opposition. The reasons for the opposition are many and diverse. J. B. Say, an influential French economist, strongly opposed the use of mathematical and statistical tools in economics. In his Traité d' Economie Politique (1803) he stated:[3]

> Political economy is like mathematics in that they are both based on abstraction. However, the important difference is that in mathematics one deals in magnitudes, while in political economy one deals in values which are dependent on the action of faculties, needs and will of man and so belong to the domain of morals. Consequently, it is superfluous to apply the formulae of Algebra to variables in Political Economy because none of these variables are susceptible to rigorous evaluation. (Say, Introduction)

On a different occasion he returned to the subject:

> All attempts to reduce political economy to mathematical calculations (had been) misleading, for the simple reason that this discipline called upon human will, human needs and faculties, which did not provide sufficiently precise data to constitute the basis of calculation. (Menard, 528)

Most of these early objections appear to be based on the notion that applying mathematics in economics is tantamount to quantifying the moral elements of social life. Consider the arguments of G. B. Venturi, an eighteenth century writer belonging to the Milanese School,[4] who made relatively extensive use of mathematics in his own economic analysis but issued the following warning to fellow members of the profession:

> We hope that these reflections will serve as a shining example to everyone of the danger one runs when one wants to use mathematics in fields outside the realm of nature and one wants to express with lines and analytical symbols moral quantities which depend on a thousand factors and which are not at all susceptible of any exact measurement. (Theocharis, 34)

Attempts to meausre preferences per se are not in vogue today--at best we study the influence of revealed preferences, i.e., choices--and it would perhaps have been wise of the early mathematical economists if they had simply moved their attention to the subject of production where measurement is feasible. Some of the early successful applications of mathematics arose in fields where the question of measurement was relatively unproblematic, such as the Quantity Theory of Money or in applications to insurance. It may be no accident that the first acceptable use of algebraic formulae in economics, e.g., to deduce marginal productivity, was made by the Prussian Junker, von Thünen, to the tangible issue of agricultural production and marketing.

Thus, early attempts by a few authors in the nineteenth century to use mathematical analysis in economics were unsuccessful because their views and methodology were in discord with those of the dominant school of the day. This was true in France, Germany and in England.

Cournot was passed over in France, Gossen in Germany and the Whewell group in England. In Professor Seligman's words,

> The edifice erected by Ricardo, and elaborated by McCulloch and Mil, became so solid and so stable that it could not be shaken by any current or gust of criticism or opposition. . . . Discouraged by their reception, most of these writers turned to other lines of activity. (Seligman, 534-535)

II.C. Mathematical Training or Lack Thereof of Earlier Economists

One important reason for the inability to incorporate mathematics successfully was a lack of clarity in the basic concepts most frequently used--demand and supply. How were these to measured? What did the phrase "an increase in demand" mean? John Stuart Mill is commonly believed to have introduced some clarity into this subject but the following complaint from J. T. Graves, a Law Professor competent in mathematics, to Whewell shows that the explicit mathematical formulation led to a puzzle because of a failure to be clear about the independent variable.[5]

> J. S. Mill (Principles, vol. i, p. 528) seems to think that he does much by making Demand and Supply respectively Functions of the Price.--(I am translating his explanation into symbolical language)--so that
>
> $$D = \phi(P) \text{ and } S = \psi(P)$$
>
> and by supposing P to be determined implicitly by the equation
>
> $$\phi(P) = \psi(P),$$
>
> the market price being that which makes demand and supply equal.
>
> Of $\phi(P)$ we are told that it is a function which increases as P diminishes, while $\psi(P)$ increases with P.
>
> This is certainly not at all what is meant in common acceptation, however vague, by Supply and Demand, which are not regarded as quantities varying during the "haggling of the market," but as elements, fixed for a time, which determine the result of that haggling.

The most important contributors to twentieth century mathematical economics--Paul Samuelson, Jan Tinbergen, Wassily Leontief, Tjalling Koopman, George Dantzig, Kenneth Arrow, Gerard Debreu, and many of the founding members of the Econometric Society--were all formally trained in mathematics and in some cases even in physics. Could it be that one factor behind the retarded progress of mathe-matization of economics was the inadequate mathematical training of the early leaders in economics or perhaps an unwillingness to use whatever mathematics they knew? This was certainly true for the founders of the Classical school--Adam Smith, Ricardo, Malthus, Marx, and J. S. Mill. There is very little mathematics to be found in <u>The Wealth of Nations</u>. Although Smith had some mathematical training, there is no reason to believe that he was sufficiently well-versed in mathematics to attempt applications to economics. Similarly, Ricardo--a stock broker by profession--lacked a good background in mathematics. O'Brien comments on Ricardo, the theoretical economist: "As a pure theorist his lack of mathematical training occasionally led him astray" (O'Brien, 270). Indeed, Whewell used Ricardo's inability to see the boundedness of certain series of numbers as a demonstration of the utility of applying mathematics. Ricardo's friend and fellow economist, Malthus, also lacked confidence in his mathematical skills despite the fact that he was the second wrangler at Cambridge, and despite the rhetorical effectiveness of the arithmetic and geometric series in his early essay on population. In 1829 when Malthus received the first paper on mathematical economics written by Whewell, he wrote back, "I have looked it over with great interest; but I am

ashamed to say that, never having been very familiar with the present algebraic notation, and for a great many years having been quite unaccustomed to using it, I cannot follow you as I could wish. . . ."[6] The writings of Marx contained numerical illustrations and formulae but little to indicate a facility with higher mathematics.[7]

We also find that the eighteenth and nineteenth century writers who successfully used mathematics as an analytical tool in their economic writings--Bernoulli, the French Engineers, Canard, Cournot, Walras, Wicksell, Marshall, Pareto, Edgeworth, and Fisher to name the most prominent--were all trained as mathematicians.[8] The twentieth century tradition of teaching advanced mathematics to scores of our graduate students in economics must have been an important catalyst in the revolutionary growth in mathematical economics in this century. To conclude this point, we find support for the thesis that imprecise economic concepts combined with the lack of mathematical training on the part of the leaders of political economy was an important discouraging factor in the mathematization of economics during the early years.

II.D. The Underdevelopment of Mathematics

A fourth factor worth investigating is the relative "underdevelopment of mathematics" itself before the twentieth century. We suggest that one reason the progress of mathematical economics was retarded was the unavailability of the tools that are most useful to this field. For a social science such as economics to gain from mathematics, it is not only the overall of sophistication of mathematics

that is relevant, but it is also important that specific tools which are useful to economics--dynamic analysis, the notion of functions, and advanced calculus--be available to be borrowed and applied.

Mathematical economists will generally agree that the most important developments in this area include multivariate calculus, mathematical programming and its extensions such as control theory, convex methods, game theory, and finally the use of modern statistical analysis to test various economic hypotheses (econometrics). Almost all these concepts were unavailable to economists before the twentieth century.

Consider the functional form, which is perhaps the most basic mathematical concept in modern economics. Since economics is mostly about relationships among variables, being able to express these relationships in the functional form of f(x) has helped immensely in the clarifying of the language of economics and in the furthering of analysis. However, the use of the functional form is relatively recent. Though it will come as a surprise to most students of economics, the earlier quoted critique of J. S. Mill's formulation shows that it took the profession a long time to clarify elementary concepts involving variables, such as the notion of demand and supply as a relationship between independent prices and dependent quantities. Although eighteenth and early nineteenth century economists frequently referred to "vent," "demand" and supply, there is little evidence to suggest that the functional relationships of demand and supply were understood in the modern sense. It was much later that these concepts were clarified and rigorously defined in economics.[9]

Why did mathematics not come to the rescue when economics was struggling to deal with these basic concepts involving relationships between variables? Perhaps we should first determine if the mathematicians themselves were clear about the functional form and had developed appropriate symbols to deal with such relationships. In his elegant series of lectures on great moments in mathematics, Howard Eves states that the modern concept of functions and the elaborate branch of mathematics known as "function theory" is almost completely a twentieth century development. (Eves, 153-55) The word "function" was first coined by Leibnitz in 1694 to denote any quantity connected with a curve such as the slope of the curve. Later the concept was refined in the hands of Bernoulli (1718) and Euler (1707-1783). The notation f(x) entered mathematics in about 1734 with A. C. Clairmont and Leonhard Euler. In 1837, L. Dirichlet arrived at the present definition, which stresses the basic idea of a relationship between two sets of numbers. Modern set theory, a twentieth century development, has revolutionized the concept of functions. Today, a functional relationship is nothing but a special kind of subset of the Cartesian product set A x B. According to another perceptive author, Salomon Bochner, functions are a distinguishing attribute of modern mathematics, perhaps the most profoundly distinguishing of all. (Bochner, 257) He goes on to say that in its innermost structure Greek mathematics was a mathematics entirely without functions and without any orientation towards functions.

It may be mentioned here that both Whewell and Cournot, accomplished mathematicians who made substantial contributions to

mathematical economics in the nineteenth century, had tried to provide a general equilibrium framework but found that the mathematics available to them to be insufficiently developed to permit the formulation of a general equilibrium model.[10]

Not only were many of the appropriate mathematical concepts lacking between 1700-1850, it is also true that some available concepts, such as the calculus, were considered highly questionable. The invention of the calculus faced mathematicians with a considerable dilemma. The power of the new tool was undeniable, but it was not at all clear how the foundations of the subject were to be made rigorous. Bishop Berkeley poked fun at the most popular way of setting up the derivative through infinitely small quantities or infinitesimals by calling them the "ghosts of departed quantities." It was not until the early nineteenth century, with the works of Cauchy and of Weirstrass that the calculus was finally set up on a firm footing. The readiness of mathematicians to use the calculus despite widespread unease about its foundations simply because it was so convenient and powerful has led Judith Grabiner to ask "Is Mathematical Truth Time-Dependent?" (Grabiner, 1981) Since several economists of this period did have training in geometry, with the high Greek standards of rigor, it is possible that they would have been turned off by the perplexing lack of rigor of seventeenth century mathematics.

The dominance of the English over continental economics until the very end of the nineteenth century is unfortunate because the English universities were not only dominated by a non-mathematical philosophy--the cult of the gentleman--some of the mathematics that was produced

was purely formal and lacking in any mathematical insight. Heaviside, for example, produced a formal calculus of operators which ignored problems posed by divergent series and focussed solely on the manipulation of symbols. A. F. Monna comments appropriately that "It is a symbolic method without giving insight in what is going on." (Monna, 1973, 84)

The axiomatic method is not new. According to some mathematicians, the most outstanding contribution of ancient Greeks was their organization of mathematics by the axiomatic method. However, the Greek axiomatic method was subjected to general neglect until the nineteenth century, when the work of a number of mathematicians gradually sharpened and refined it. The Grundlagen der Geometrie written by Hilbert in 1899 played a significant role in revamping the ancient Greek axiomatics from material axioms to the formal axioms which are used by mathematicians and economists today. Irving Fisher was later to make a pointed remark about the difference between using mathematical methods and acquiring the mathematical spirit. The axiomatic method is perhaps the most determined driving force in urging the acquisition of the mathematical spirit. Whewell presented six "axioms" in his presentation of Ricardo's model, however three of his axioms can be deduced from the other assumptions and a fourth is redundant. (Rashid, 1977, 384) Once again we see an important mathematical tool available to the economic theorist today was not available before the twentieth century.

III. CONCLUSION

Joseph Schumpeter asked why the brilliant contributions of several mathematical economists in the nineteenth century--Cournot, Gossen, Whewell--were unsuccessful in impacting the development of the subject. According to Schumpeter, since the marginal revolution was not entirely a mathematical revolution, the non-mathematical majority of the profession came to believe that except for a few refinements, the contribution of mathematical analysis to political economy has been insignificant. Secondly, the leaders of the profession--economists in or past their middle ages--were poorly trained in mathematics and therefore wanted to spare themselves the trouble of learning what by all accounts was a difficult technique with dubious benefits. Some of them attempted to justify their lack of interest in learning mathematics by becoming its critics. In Schumpeter's words:

> Not less understandably, they rationalized this attitude and produced in its defense a number of methodological arguments such as that the attempt to apply mathematics, the tools of physics, to the social sciences was a mistake on logical principle, . . . (Schumpeter, p. 957).

While there is considerable truth in Schumpeter's analysis, we have argued that one can go further than these ad hominem arguments.

This paper has identified and discussed several factors that may have contributed to retarding the mathematization of economics before the twentieth century. These include the early opposition to such attempts by many important members of the profession and the prevailing anti-mathematical climate of the times, the discouraging effects of the few early missteps, the fact that earlier economists were inadequately trained in mathematics, and the fact that many of the

mathematical concepts and symbols important to economics were unavailable to the earlier economists grappling with inherently mathematical notions in economic theory. While each of the above points has been presented as an independent unit, it will have occurred to the reader that such a division is purely one of convenience. For example, if the early economists had been more mathematically knowledgeable they could have distinguished between ordinal and cardinal variables and obviated much needless debate about measurement; similarly, if matrix algebra had been available in the late eighteenth century, a discerning Physiocrat may have put Quesnay's *Tableau* in modern input-output form. The relationship between the growth of economics and mathematics is best indicated by that over-used word, "dialectical."

The major stumbling block which frustrated earlier economists was multivariate functional relationships. They were frequently unable to deal with functions with two or more independent variables. The solution of most economic problems involving a single variable can be as satisfactorily obtained with diagrams as with algebra. To attack more significant problems requires mathematics involving several variables; both Whewell and Cournot were unable to make any progress on general equilibrium issues because of the lack of linear algebra or multivariate calculus. The common criticism that economic questions depended upon a multitude of factors could at least have been satisfactorily met in such formulations. Why did they not arise?

If any one feature is to be singled out as having slowed down the formation of mathematical economics it is probably the persistence of certain philosophical presuppositions of English university thought.

The refusal to view other disciplines as instrumental, without need of philosophical guidance, was repugnant and we have seen how repeatedly it led to negative remarks on the use of mathematics in economics. It is surprising, but nonetheless true, that a similar ontological status granted to Euclidean geometry appears to have hindered the growth of linear algebra. In his early papers Cayley wrote apologetically about n-dimensional space--how could one conceive of more than three dimensions? How far the rise of non-Euclidean geometry freed the general milieu from such philosophical constraints is a topic worth further study.[11]

The fact that economics was rapidly mathematized in the twentieth century suggests the great usefulness of mathematics to many areas in economics. This need for mathematical tools was recognized early on. However, this recognition was a necessary but not a sufficient condition for the widespread use of mathematics in economics. The lack of the "sufficient conditions precluded a more accelerated development of mathematical economics prior to the twentieth century.

NOTES

[1]The list of scholars who have explored the history of mathematical economics includes Schumpeter (1954), Robertson (1949), Theocharis (1983), and Henderson (1985). Schumpeter and Robertson conclude that no school of mathematical economics existed prior to the twentieth century although there are isolated contributions--many of extra-ordinary quality. Theocharis identifies several schools, but except for Cournot, the mathematics employed was relatively rudimentary. Henderson identifies the Whewell group that existed in Cambridge during the first half of the nineteenth century. Once again their influence on the general direction of the field was minor and the level of mathematics used low, by today's standards.

[2]We do not intend to imply any judgment on Canard, who has been recently defended by Larson (1989), but rather to focus on the reception of Canard by contemporaries.

[3]Menard (1980) argues that the three most important French economists of the nineteenth century--Say, Cournot, and Walras--all opposed the use of statistical methods in economics for one reason or another.

Say spoke against the use of mathematics and formulae in economics because they are "most evidently inapplicable to Political economy and form the most dangerous of abstractions." His opposition to statistics can be understood from the following passage he wrote: "when I see that there has been no detestable undertaking that has not been supported and determined by arithmetical calculations, I am lead to believe that it is figures which are the downfall of the sate." (Menard, 527).

[4]Theocharis uses the term 'Milanese School' to refer to the group of eighteenth and nineteenth century economists centered in Milan, who used mathematics in their economic writings. Venturi was a Professor of Mathematics and Philosophy at the University of Modena.

[5]Whewell papers, Trinity College, Cambridge, J. Graves to Whewell, January 9, 1849. We are grateful to the keepers of the Whewell Papers for use of this material.

[6]Quoted in Henderson, p. 426.

[7]Professor Baumol has informed us that Marx produced mathematical notebooks said to contain lagrange multipliers. However, these are yet to be published by Moscow where they are being edited. Zauberman (1975) also informs us that Marx, according to a Russian translation of his work, had written that a social science is elevated to the status of a science only when it becomes susceptible to mathematics.

[8]See Blaug, p. 311 and Schumpeter, p. 956.

[9]According to Blaug, Cournot was the first writer to define and draw a demand function in 1938. See Blaug, p. 333. Also, for a long time there was a confusion about the dependent variable in the demand function.

[10]Cournot had concluded that "for a complete and rigorous solution of the problems relative to some parts of the economy system, it [is] indispensable to take the entire system into consideration." See Cournot (1838), p. 127. For Whewell see Rashid (1977).

[11]Marshall is only the most well-known of mathematically competent economists, who opted not to explicitly employ mathematical techniques in their economic analysis. Others in this group include, Sidgwick, Ingram, and Lexis. These economists, for various reasons, were hostile to the use of mathematics in economics. Marshall's opposition to the explicit use of mathematical language is legend. He wrote in a letter to A. L. Bowley in 1906:

> I had a growing feeling in the later years of my work at the subject that a good mathematical theorem dealing with economic hypotheses was very unlikely to be good economics: and I went more and more on the rules--(1) Use mathematics as a shorthand language, rather than as an engine of inquiry. (2) Keep to them until you are done. (3) Translate into English. (4) Then illustrate by examples what are important in real life. (5) Burn the mathematics. (6) If you can't succeed in (4), burn (3). This last I did often. (Pigou, 427).

[12]See the fascinating remarks by Joan Richards in Davis and Hirsh (1986), 203-217, as well as Richards (1988).

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